Academic Team
Math Must Knows

Pythagorean Triples

Almost certainly, the most important things to know are the basic sets of integers that satisfy the Pythagorean Theorem \((a^2 + b^2 = c^2)\) and could be the side lengths of a right triangle. These are called Pythagorean Triples and the simplest ones are 3-4-5, 5-12-13, 7-24-25, and 8-15-17. Note that any multiple of a Pythagorean Triple is also a Pythagorean Triple so that 6-8-10, 15-20-25, and 300-400-500 are also ones by virtue of 3-4-5 being one.

Solids
Teams should be able to calculate the volume and surface area of simple geometric figures including the sphere, cone, cylinder, pyramid, hemisphere, prism, and parallelepiped.

Plane Figures
Teams should be able to calculate the areas of triangles (especially equilateral triangles), trapezoids, parallelograms, rhombi, and circles using different angles and lengths.
Quadratics

Quadratics are polynomials of degree 2. The graph of a quadratic equation will be in the shape of a parabola that opens straight up (if the coefficient on the $x^2$ term is positive) or straight down (if that coefficient is negative). It is possible to find the roots of a quadratic by graphing it, factoring it, completing the square on it, or using the quadratic formula (itself derived by completing the square) on it. If the quadratic is in the form $ax^2 + bx + c$, then the expression $b^2 - 4ac$, which appears in the quadratic formula, is called the discriminant. If the discriminant is positive, the quadratic will have two real roots; if the discriminant is zero, the quadratic will have one real root (said to have a multiplicity of 2); and if the discriminant is negative, the quadratic will have two non-real complex roots (and if the coefficients of the quadratic are real numbers, the complex roots will be conjugates of each other).

Exponential Functions

Exponential functions are those of the form $f(x) = b^x$, where $b$ (called the base) is a positive number other than 1. Exponential functions are used to model unrestricted growth (such as compound interest, and animal populations with unlimited food and no predators) and decay (such as radioactive decay). The phrase “the exponential function” refers to the function $f(x) = e^x$, where $e$ is a specific irrational number called Euler’s number, about equal to 2.718. Exponential functions have the interesting property that their derivatives are proportional to themselves.

The work of Isaac Newton (1643-1727, English) in pure math includes generalizing the binomial theorem to non-integer exponents, doing the first rigorous manipulation with power series, and creating “Newton’s method” for the finding roots. He is best known, however, for a lengthy feud between British and Continental mathematicians over whether he or Gottfried Leibniz invented calculus (whose differential aspect Newton called “the method of fluxions”). It is now generally accepted that they both did, independently.

Know these mathematicians:

Newton, Euclid, Gauss, Archimedes, Leibniz, Fermat, Euler, Godel, Wiles, Hamilton.
Euclid (c. 300 BC, Alexandrian Greek) is principally known for the Elements, a textbook on geometry and number theory, that was used for over 2,000 years and which grounds essentially all of what is taught in modern high school geometry classes. Euclid is known for his five postulates that define Euclidean (i.e., "normal") space, especially the fifth (the "parallel postulate") which can be broken to create spherical and hyperbolic geometries. He also proved the infinitude of prime numbers.

Carl Friedrich Gauss (1777-1855, German) is considered the "Prince of Mathematicians" for his extraordinary contributions to every major branch of mathematics. His Disquisitiones Arithmeticae systematized number theory and stated the fundamental theorem of arithmetic. He also proved the fundamental theorem of algebra, the law of quadratic reciprocity, and the prime number theorem. Gauss may be most famous for the (possibly apocryphal) story of intuiting the formula for the summation of an arithmetic series when given the busywork task of adding the first 100 positive integers by his primary school teacher.

Archimedes (287-212 BC, Syracusan Greek) is best known for his "Eureka moment" of using density considerations to determine the purity of a gold crown; nonetheless, he was the preeminent mathematician of ancient Greece. He found the ratios between the surface areas and volumes of a sphere and a circumscribed cylinder, accurately estimated pi, and presaged the summation of infinite series with his "method of exhaustion."

Gottfried Leibniz (1646-1716, German) is known for his independent invention of calculus and the ensuing priority dispute with Isaac Newton. Most modern calculus notation, including the integral sign and the use of d to indicate a differential, originated with Leibniz. He also invented binary numbers and did fundamental work in establishing boolean algebra and symbolic logic.

Pierre de Fermat (1601-1665, French) is remembered for his contributions to number theory including his "little theorem" that $a^p - a$ will be divisible by p if p is prime. He also studied Fermat primes (see below) and stated his "Last Theorem" that $x^n + y^n = z^n$ has no solutions if x, y, and z are positive integers and n is a positive integer greater than 2. He and Blaise Pascal founded probability theory. In addition, he discovered methods for finding the maxima and minima of functions and the areas under polynomials that anticipated calculus and inspired Isaac Newton.

Fermat primes: (those of the form $2^{2^n} + 1$)

Leonhard Euler (1707-1783, Swiss) is known for his prolific output and the fact that he continued to produce seminal results even after going blind. He invented graph theory with the Seven Bridges of Königsberg problem and introduced the modern notation for e, the square root of -1 (i), and trigonometric functions. Richard Feynman called his proof that $e^{i\pi} = -1$ "the most beautiful equation in mathematics" because it linked four of math's most important constants.
Kurt Gödel (1906-1978, Austrian) was a logician best known for his two incompleteness theorems proving that every formal system that was powerful enough to express ordinary arithmetic must necessarily contain statements that were true, but which could not be proved within the system itself.

Andrew Wiles (1953-present, British) is best known for proving the Taniyama-Shimura conjecture that all rational semi-stable elliptic curves are modular. This would normally be too abstruse to occur frequently in quiz bowl, but a corollary of that result established Fermat's Last Theorem.

William Rowan Hamilton (1805-1865, Irish) is known for extending the notion of complex numbers to four dimensions by inventing the quaternions, a non-commutative field with six square roots of -1: ±i, ±j, and ±k with the property that ij = k, jk = i, and ki = j.

The period of one of these objects is proportional to the square root of its length and independent of its mass. Léon Foucault [lay-awn foo-KOH] used one to demonstrate the (*) rotation of the Earth.

For 10 points—what objects with a freely swinging mass are sometimes used to keep time in clocks?

answer: pendulums (or pendula; accept simple pendulum, ideal pendulum, or simple gravity pendulum)

In 1882 Ferdinand von Lindemann proved that this number is transcendental. Ptolemy [TAH-luh-mee] estimated it as 377 over 120; other approximations include the square root of 10 and (*) 22/7.

For 10 points—name this constant, the ratio of the circumference to the diameter of a circle, equal to about 3.14.

answer: pi

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Pencil and paper ready. Debbie earned 51 dollars in tips yesterday, which was 15 percent of the pre-tip value of her customers' orders. To find the value of her customers' orders, one can either use an algebraic equation or ratios.

For 10 points—find the pre-tip value of the orders.

answer: $340.00 \ [0.15x = 51, \ so \ x = \frac{51}{0.15} = \frac{51}{3/2} = \frac{51 \times (2/3)}{1} = \frac{51}{1} \times \frac{2/3}{1} = 17 \times 20 = 340]$; alternatively, since 15% = $51, 15\% \times (2/3) = 51 \times (2/3), \ so \ 10\% = 34, \ and \ 100\% = 340.$

Jan 25-9:55 AM

The "standard form" for this shape's equation is \( ax + by = c \).

For 10 points each—A. Name this figure in the plane.

answer: line(s)

B. \( y = mx + b \) ["Y equals M X plus B"] expresses a line in this common form, named for what \( m \) and \( b \) represent.

answer: slope-intercept form (accept slope/y-intercept form)

C. This type of line cannot be expressed in slope-intercept form, because its slope is undefined.

answer: vertical line (prompt on answers mentioning "x equals")

Jan 25-10:03 AM

Pencil and paper ready. Greg used 5 and two-fifths cups of sugar to bake 3 dozen cookies, and he needs to know how many cups are needed to make 12 dozen cookies. By converting 5 and two-fifths into an improper fraction and using a ratio, he computes (*)—for 10 points—how many cups to use?

answer: 21 3/5 cups or 108/5 cups or 21.60 cups \([5\frac{2}{5}] = 5 \times \frac{12}{3} = x/12, \ so \ (27/5) = 3x, \ and \ x = 4 \times (27/5) = 108/5 = 21 \ 3/5\]

Jan 25-10:09 AM

Pencil and paper ready. A gardener mowed two-fifths of a lawn on Monday, and then mowed three-quarters of the remaining portion on Tuesday.

For 10 points each—A. What proportion of the entire lawn did the gardener mow on Tuesday?

answer: 5/20 or 0.25 or 25% [After Monday, 1 - (2/5) = 3/5ths of the lawn was unmowed; 3/4ths of that fraction is (3/5) × (3/4) = 9/20.]

B. What proportion of the entire lawn was still unmowed after Tuesday?

answer: 3/20 or 0.15 or 15% [Between Monday and Tuesday, he mowed \((2/5) + (9/20) = (8/20) + (9/20) = 17/20ths\) of the lawn, leaving 1 - (17/20) = 3/20ths unmowed.]

C. If the lawn's total area is 1,800 square feet, how many square feet were still unmowed after Tuesday?

answer: 270 square feet \([1800 \times (3/20) = 90 \times 3 = 270]\)

Jan 25-10:09 AM

On the number line, multiplying by \(-1\) ["negative one"] can be thought of as rotating by this many degrees, which is equal to \( \pi \) radians [RAY-dee-unz]. The interior angles of a triangle add up to this many degrees, and so do a pair of supplementary angles. For 10 points—give this value equal to half of a complete turn.

answer: 180 degrees (accept pi radians before "degrees")

Jan 25-10:09 AM

Pencil and paper ready. A circle has a radius of 10. For 10 points each—A. What is the length of the longest chord of the circle?

answer: 20 [The longest chord of a circle is the diameter, which is twice the radius; \( 2 \times 10 = 20 \).]

B. What is the length of an arc of the circle that subtends a central angle of 90 degrees?

answer: 5 \( \pi \) [do not accept or prompt on "\( \pi \)""] [The complete circumference of the circle is \( C = 2 \times \pi \times r = 2 \times \pi \times 10 = 20 \pi \), so the length of the arc is \( 20 \pi \times (90/360) = (20 \pi \times (1/4) = 5 \pi \).]

C. What is the length of the chord that, at its closest approach, is a distance of 6 away from the center of the circle? You have 10 seconds.

answer: 16 [The chord, a line segment connecting the chord to the center at its closest approach, and 2 radii will create two, congruent 6-8-10 right triangles; \( 8 + 8 = 16 \).]
His "rule of signs" gives an upper bound on the number of roots of one type of expression. For 10 points each—A. Name this French mathematician and philosopher known for a namesake coordinate system.

answer: René Descartes [reh-nay day-"cart"] (or Renatus Cartesius; prompt on "Cartesian (coordinate system)"

B. The rule of signs applies to this type of expression, a combination of variables and constants combined by addition, subtraction, and multiplication. Examples include $3x^2 + 5xy - 2y$.

answer: polynomial(s) [pah-lih-N-OH-mee-ul] (accept trinomial)

C. The coefficients of the polynomial expansion of $(x+1)^3$ ["x plus one, quantity cubed"] can be found on the third row of the triangle named after this other French mathematician.

answer: Blaise Pascal [pass-kal] (accept Pascal's triangle)