Binomial vs. Geometric

Chapter 8
Binomial and Geometric Distributions
## Binomial vs. Geometric

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Are Random Variables and Binomial Distributions Linked?

X = number of people who purchase electric hot tub

<table>
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<tr>
<th>X</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>P(X)</td>
<td>.216</td>
<td>.432</td>
<td>.288</td>
<td>.064</td>
</tr>
</tbody>
</table>

GGG  (.6)(.6)(.6)  EEG  (.4)(.4)(.6)
EGG  (.4)(.6)(.6)  GEE  (.6)(.4)(.4)
GEG  (.6)(.4)(.6)  EGE  (.4)(.6)(.4)
GGE  (.6)(.6)(.4)  EEE  (.4)(.4)(.4)
Combinations

Formula: \( \binom{n}{k} = \frac{n!}{k!(n-k)!} \)

Practice:

1. \( \binom{6}{4} = \frac{6!}{4!(6-4)!} = \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{4 \cdot 3 \cdot 2 \cdot 1 \cdot 2 \cdot 1} = \frac{6 \cdot 5}{2 \cdot 1} = 15 \)

2. \( \binom{8}{5} = \frac{8!}{5!3!} = \frac{8 \cdot 7 \cdot 6 \cdot 5!}{5! \cdot 3 \cdot 2 \cdot 1} = 56 \)
### Developing the Formula

<table>
<thead>
<tr>
<th>Outcomes</th>
<th>Probability</th>
<th>Rewritten</th>
</tr>
</thead>
<tbody>
<tr>
<td>$O^cO^cO^c$</td>
<td>$(.75)(.75)(.75)$</td>
<td>$\binom{3}{0}(.25)^0(.75)^3$</td>
</tr>
<tr>
<td>$OO^cO^c$</td>
<td>$(.25)(.75)(.75)$</td>
<td>$\binom{3}{1}(.25)^1(.75)^2$</td>
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<tr>
<td>$O^cO^cO^c$</td>
<td>$(.25)(.75)(.75)$</td>
<td>$\binom{3}{2}(.25)^2(.75)^1$</td>
</tr>
<tr>
<td>$O^cO^cO$</td>
<td>$(.25)(.25)(.75)$</td>
<td>$\binom{3}{3}(.25)^3(.75)^0$</td>
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<td>$OOO^c$</td>
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<tr>
<td>$P(X)$</td>
<td>.4219</td>
<td>.4219</td>
<td>.1406</td>
<td>.0156</td>
</tr>
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Developing the Formula

\[ P(X = k) = \binom{n}{k} p^k (1 - p)^{n-k} \]

- \( n \) = number of observations
- \( p \) = probability of success
- \( k \) = given value of variable

Rewritten

\[
\begin{align*}
\binom{3}{0} (0.25)^0 (0.75)^3 \\
\binom{3}{1} (0.25)^1 (0.75)^2 \\
\binom{3}{2} (0.25)^2 (0.75)^1 \\
\binom{3}{3} (0.25)^3 (0.75)^0
\end{align*}
\]
Working with probability distributions

- State the distribution to be used
- Define the variable
- State important numbers
  - Binomial: n & p
  - Geometric: p
Twenty-five percent of the customers entering a grocery store between 5 p.m. and 7 p.m. use an express checkout. Consider five randomly selected customers, and let $X$ denote the number among the five who use the express checkout.

\[
\text{binomial} \quad n = 5 \quad p = .25
\]

$X = \# \text{ of people use express}$
What is the probability that two used express checkout?

binomial \quad n = 5 \quad p = .25

\[ X = \# \text{ of people use express} \]

\[ P \quad X = 2 = \binom{5}{2} \cdot .25^2 \cdot .75^3 = .2637 \]
What is the probability that at least four used express checkout?

binomial \quad n = 5 \quad p = .25

\[ X = \# \text{ of people use express} \]

\[
P(X \geq 4) = \binom{5}{4} .25^4 .75^1 + \binom{5}{5} .25^5
\]

\[= .0156\]
“Do you believe your children will have a higher standard of living than you have?” This question was asked to a national sample of American adults with children in a *Time/CNN* poll (1/29,96). Assume that the true percentage of all American adults who believe their children will have a higher standard of living is .60. Let X represent the number who believe their children will have a higher standard of living from a random sample of 8 American adults.

\[
\text{binomial} \quad n = 8 \quad p = .60 \\
X = \# \text{ of people who believe}...
\]
Interpret $P(X = 3)$ and find the numerical answer.

binomial $\quad n = 8 \quad p = .60$

$X = \# \text{ of people who believe}$

The probability that 3 of the people from the random sample of 8 believe their children will have a higher standard of living.

$$P \ X = 3 \ = \ \binom{8}{3} .6^3 .4^5 \ = .1239$$
Find the probability that none of the parents believe their children will have a higher standard.

\[
\text{binomial} \quad n = 8 \quad p = 0.60
\]

\[
X = \# \text{ of people who believe}
\]

\[
P(X = 0) = \binom{8}{0} \cdot 0.6^0 \cdot 0.4^8 = 0.00066
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## Developing the Geometric Formula

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<th>( P( X = n ) = (1 - p)^{n-1} p )</th>
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<tr>
<td>1</td>
<td>1/6</td>
<td>( (5/6)(1/6) )</td>
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<td>( (5/6)(5/6)(1/6) )</td>
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<td>4</td>
<td>( 5/6 )</td>
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The Mean and Standard Deviation of a Geometric Random Variable

If $X$ is a geometric random variable with probability of success $p$ on each trial, the expected value of the random variable (the expected number of trials to get the first success) is

$$
\mu = \frac{1}{p} \quad \sigma = \sqrt{\frac{1-p}{p^2}}
$$
Suppose we have data that suggest that 3% of a company’s hard disc drives are defective. You have been asked to determine the probability that the first defective hard drive is the fifth unit tested.

geometric \[ p = .03 \]

\[ X = \# \text{ of disc drives till defective} \]

\[ P \ X = 5 \ = \ .97^4 \cdot .03 = .0266 \]
A basketball player makes 80% of her free throws. We put her on the free throw line and ask her to shoot free throws until she misses one. Let $X = \text{the number of free throws the player takes until she misses.}$

\[
\text{geometric} \quad p = 0.20 \\
X = \# \text{ of free throws till miss}
\]
What is the probability that she will make 5 shots before she misses?

geometric \hspace{1cm} p = .20

\[ X = \# \text{ of free throws till miss} \]

\[ P \ X = 6 = .80^5 \cdot .20 = .0655 \]

What is the probability that she will miss 5 shots before she makes one?

geometric \hspace{1cm} p = .80

\[ Y = \# \text{ of free throws till make} \]

\[ P \ Y = 6 = .20^5 \cdot .80 = .00026 \]
What is the probability that she will make at most 5 shots before she misses?

geometric \hspace{0.5cm} p = .20

X = # of free throws till miss

\[ P \ X \leq 6 = .20 + .80 \cdot .20 + .80^2 \cdot .20 \]
\[ + .80^3 \cdot .20 + .80^4 \cdot .20 + .80^5 \cdot .20 \]

\[ = .7379 \]